New Properties of the Odd Weibull Inverse Topp-Leone Cumulative Distribution Function

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joint work with
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Introduction

Definition 1. (***Almetwally (2021)***)

The odd Weibull inverted Topp–Leone (OWITL) distribution is associated with the cdf given as

$$F(t;\alpha,\lambda,\delta) = 1 - e^{-\lambda[(1+t)^{2\delta}(1+2t)^{-\delta}-1]^{\alpha}}, \quad t > 0, \alpha, \lambda, \delta > 0. \tag{1}$$

• $\lambda = 1$ OWITL distribution consist with two parameter modified Kies inverted Topp-Leone (MKITL) distribution (*Almetwally, Alharbi, Alnagar, and Hafez (2021)*)

Purposes:

• study one of the important characteristics "saturation"of cumulative function to the horizontal asymptote with respect to Hausdorff metric with some estimates;

• consider a new adaptive model with “polynomial variable transfer” with simulation study to ”COVID-19 data”.

Introduction

Definition 2.

The shifted Heaviside step function is defined by

$$h_{t_0}(t) = \begin{cases} 0 & \text{if } t < t_0, \\ [0, 1] & \text{if } t = t_0, \\ 1 & \text{if } t > t_0. \end{cases}$$

Definition 3. (***Hausdorff (1962), Sendov (1990)***)

The Hausdorff distance (the H–distance) $\rho(f, g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f, g) = \max\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B||\}, \qquad (2)$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t, x)|| = \max\{|t|, |x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A - B|| = \max(|t_A - t_B|, |x_A - x_B|)$.

Some remarks

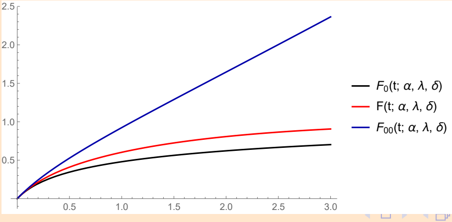
Lemma 1.

The following inequality holds

$$F_0(t;\alpha,\lambda,\delta) \leq F(t;\alpha,\lambda,\delta) \leq F_{00}(t;\alpha,\lambda,\delta),$$

where

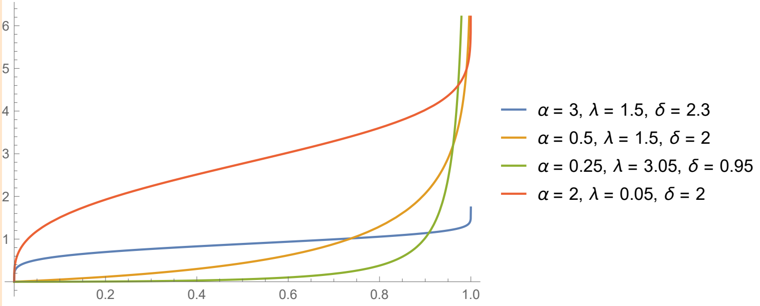
$$\begin{aligned} F_0(t;\alpha,\lambda,\delta) &= \frac{\lambda\left((t+1)^{2\delta}(2t+1)^{-\delta}-1\right)^{\alpha}}{\lambda\left((t+1)^{2\delta}(2t+1)^{-\delta}-1\right)^{\alpha}+1} \quad \text{and} \\ F_{00}(t;\alpha,\lambda,\delta) &= \lambda\left((t+1)^{2\delta}(2t+1)^{-\delta}-1\right)^{\alpha}. \end{aligned}$$



Main result

The quantile function

$$Q(u) = \left(\left(1 - \left(1 + \left(\frac{-\log(1-u)}{\lambda} \right)^{1/\alpha} \right)^{-1/\delta} \right)^{-0.5} - 1 \right)^{-1}. \quad (3)$$



The median is obtained by substituting $u = 0.5$ in (3).

Main result

$F(t_0, \alpha, \lambda, \delta) = \frac{1}{2}$, i.e. $t_0 = Q(0.5)$.

"Saturation" - d in the Hausdorff sense to the horizontal asymptote

$$F(t_0 + d, \alpha, \lambda, \delta) = 1 - d \tag{4}$$

or

$$e^{-\lambda[(1+t)^{2\delta}(1+2t)^{-\delta}-1]^{\alpha}} = d.$$

Main result

Theorem 1.

Let

$$A = 1+\frac{1}{x+1}\left(\alpha\delta\lambda\left(\frac{x}{x-1}\right)^{2\delta-1}\left(\frac{x+1}{x-1}\right)^{-\delta}\left(\left(\frac{x}{x-1}\right)^{2\delta}\left(\frac{x+1}{x-1}\right)^{-\delta} - 1 \qquad (5)$$

$$\text{with} \quad x = \left(1-\left(\left(\frac{1}{\lambda}\right)^{1/\alpha}\sqrt[\alpha]{\log(2)}+1\right)^{-1/\delta}\right)^{-0.5}$$

and $2.1A > e^{1.05}$. Then for the Hausdorff distance d between shifted Heaviside function $h_{t_0}(t)$ and the OWITL CDF function $F(t;\alpha,\lambda,\delta)$ defined by (1) the following inequalities hold true:

$$d_l = \frac{1}{2.1A} < d < \frac{\ln(2.1A)}{2.1A} = d_r.$$

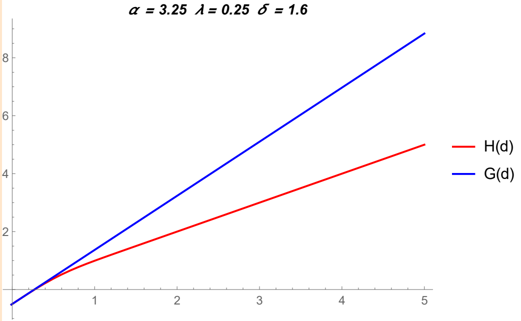
Main result

Special functions

$$G(d) = -\frac{1}{2} + Ad \tag{6}$$

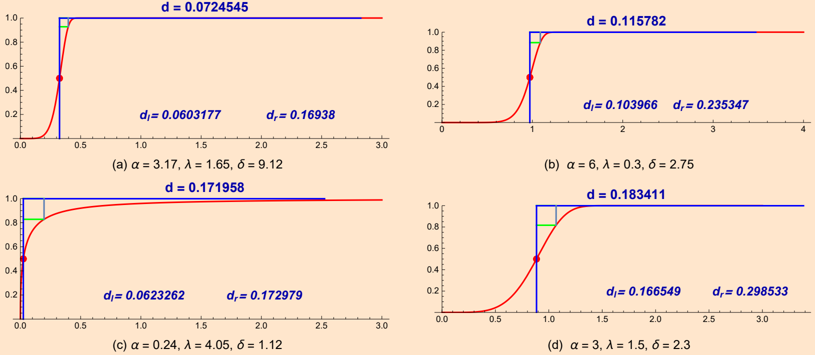
$$H(d) = F(t_0 + d, \alpha, \lambda, \delta) - 1 + d. \tag{7}$$

$$G(d) - H(d) = \mathcal{O}(d^2)$$



Computational examples

α	λ	δ	d_l	d computed by (4)	d_r
3.64	0.45	6.56	0.0768882	0.0889537	0.197249
5.39	2.95	3.12	0.0847971	0.0970375	0.209236
0.25	3.05	0.95	0.120697	0.235087	0.255211



(a) $\alpha = 3.17$, $\lambda = 1.65$, $\delta = 9.12$

(b) $\alpha = 6$, $\lambda = 0.3$, $\delta = 2.75$

(c) $\alpha = 0.24$, $\lambda = 4.05$, $\delta = 1.12$

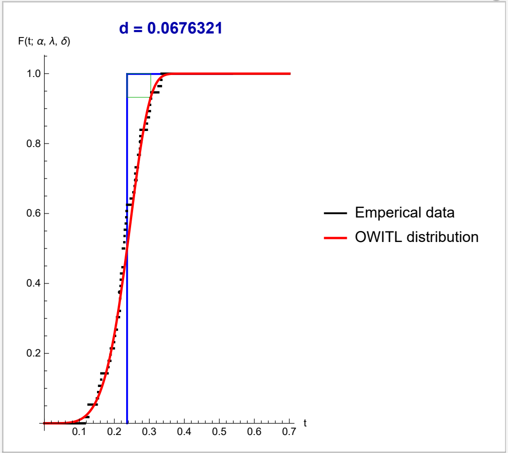
(d) $\alpha = 3$, $\lambda = 1.5$, $\delta = 2.3$

OWITL distribution for COVID-19 data of Canada

α 2.51

λ 2.2

δ 13.136



Adaptive OWITL model with "polynomial variable transfer"

Definition 4.

Consider the following new "*adaptive OWITL model with polynomial variable transfer*":

$$F^*(t;\alpha,\lambda,\delta) = 1 - e^{-\lambda[(1+f(t))^{2\delta}(1+2f(t))^{-\delta}-1]^{\alpha}},$$

$$f(t) = \sum_{i=0}^{n} a_i t^i, \quad a_0 = 0. \tag{8}$$

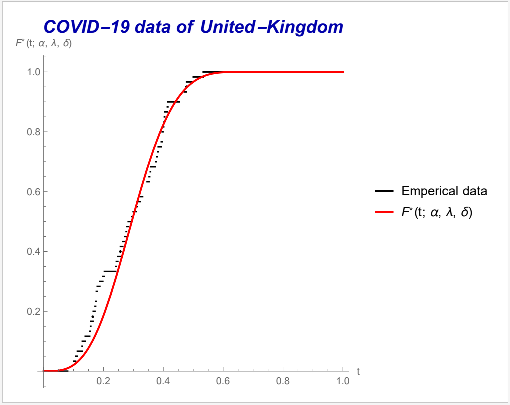
Adaptive OWITL model with "polynomial variable transfer"

Adaptive OWITL model with polynomial variable transfer

α		1.233
λ		0.7076
δ		13.136

$$f(t) = \sum_{i=0}^{n} a_i\, t^i$$

n	4
a_0	0
a_1	0.3223
a_2	4.7623
a_3	−10.8312
a_3	7.9322



Thank you for your attention!

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